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Tugas 5 Metmat (0)

3). $2 \cdot 5 \cdot 8 + 5 \cdot 8 \cdot 11 + 8 \cdot 11 \cdot 14 + \dots + 20 \cdot 23 \cdot 26$

Suku umum : $(2+3x)(5+3x)(8+3x)$

$$(10 + 21x + 9x^2)(8 + 3x)$$

$$(80 + 30x + 162x + 63x^2 + 72x^2 + 27x^3)$$

$$(27x^3 + 135x^2 + 198x + 80)$$

$$A(x)(x-1)(x-2) + B(x)(x-1) + Cx + D = 27x^3 + 135x^2 + 198x + 80$$

• $x = 0 \rightarrow D = 80$

• $x = 1 \rightarrow C + D = 440$

$$C = 360$$

• $x = 2 \rightarrow 2B + 2C + D = 1232$

$$2B + 720 + 80 = 1232$$

$$2B = 432$$

$$B = 216$$

• $x = -1 \rightarrow -6A + 2B - C + D = -10$

$$-6A + 432 - 360 + 80 = -10$$

$$-6A = -162$$

$$A = 27$$

$$\Rightarrow 27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80$$

$n=6$

$$\sum_{i=0}^n 27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80$$

$$\Delta^{-1} \left[27x^{(3)} + 216x^{(2)} + 360x^{(1)} + 80 \right]_0^7$$

$$\left[\frac{27}{4} x^{(4)} + \frac{216}{3} x^{(3)} + \frac{360}{2} x^{(2)} + 80x^{(1)} \right]_0^7$$

$$= \frac{27}{4} (7)(6)(5)(4) + \frac{216}{3} (7)(6)(5) + \frac{360}{2} (7)(6) + 80(7)$$

$$= 5670 + 15120 + 7560 + 560$$

$$= 28.910$$

$$5). 1.3.5 + 2.4.6 + 3.5.7 + \dots$$

$$\text{Suku Umum : } (x)(x+2)(x+4)$$

$$(x^2+2x)(x+4)$$

$$= x^3 + 4x^2 + 2x^2 + 8x$$

$$= x^3 + 6x^2 + 8x$$

$$A(x)(x-1)(x-2) + Bx(x-1) + Cx + D = x^3 + 6x^2 + 8x$$

$$x=0 \rightarrow D=0$$

$$x=1 \rightarrow C+D=15$$

$$C=15$$

$$x=2 \rightarrow 2B+2C+D=48$$

$$2B+30=48$$

$$2B=18$$

$$B=9$$

$$x=-1 \rightarrow -6A+2B-C+D=-3$$

$$-6A+18-15=-3$$

$$-6A=-6$$

$$A=1$$

$$\text{Suku Umum : } x^{(3)} + 9x^{(2)} + 15x^{(1)}$$

$$\sum_{i=1}^n x^{(3)} + 9x^{(2)} + 15x^{(1)}$$

$$= \Delta^{-1} x^{(3)} + 9x^{(2)} + 15x^{(1)} \Big|_1^{n+1}$$

$$= \left[\frac{1}{4} x^{(4)} + 3x^{(3)} + \frac{15}{2} x^{(2)} \right]_1^{n+1}$$

$$= \frac{1}{4} (n+1)(n)(n-1)(n-2) + 3(n+1)n(n-1) + \frac{15}{2} (n+1)n - \left[\frac{1}{4} (1)(0)(-1)(-2) + 3(1)(0)(-1) + \frac{15}{2} (1)(0) \right]$$

$$= \frac{1}{4} n(n+1) \left[n^2 - 3n + 2 + 12n - 12 + 30 \right] - 0$$

$$= \frac{1}{4} n(n+1) \left[n^2 + 9n + 20 \right]$$

7). $1 + 0 + 1 + 8 + 29 + 80 + 193 \dots$, s/d suku ke- n

1				
0	-1	2		
1	1	6	4	
8	7	14	8	4
29	21	30	16	8
80	51	62	32	16
193	113			

mulai $\Delta^3 U_x = a \cdot 2^x$, $n \geq 3$

Suku Umum : $U_x = f(x) + a \cdot 2^x$

Karena pada $\Delta^3 U_x$, nilai dari $\Delta^3 f(x) = 0$, maka dapat disimpulkan bahwa $f(x)$ merupakan polinomial derajat dua.

$$\Rightarrow U_x = Bx^2 + Cx + D + A \cdot 2^x$$

Substitusi nilai x pada tiap fungsi untuk memperoleh nilai A, B, C , dan D

$$\rightarrow x = 1 \rightarrow 1 = B + C + D + 2A \dots\dots (1)$$

$$\rightarrow x = 2 \rightarrow 0 = 4B + 2C + D + 4A \dots\dots (2)$$

$$\rightarrow x = 3 \rightarrow 1 = 9B + 3C + D + 8A \dots\dots (3)$$

$$\rightarrow x = 4 \rightarrow 8 = 16B + 4C + D + 16A \dots\dots (4)$$

► Gunakan Eliminasi dan Substitusi

Dari (1) dan (2)

$$4A + 4B + 2C + D = 0$$

$$2A + B + C + D = 1$$

$$\hline 2A + 3B + C = -1 \dots\dots (5)$$

Dari (2) dan (3)

$$8A + 9B + 3C + D = 1$$

$$4A + 4B + 2C + D = 0$$

$$\hline 4A + 5B + C = 1 \dots\dots (6)$$

Dari (3) dan (4)

$$\begin{array}{rcll} 16A + 16B + 4C + D & = & 8 \\ 8A + 9B + 3C + D & = & 1 \\ \hline 8A + 7B + C & = & 7 \dots\dots (7) \end{array}$$

Dari (6) dan (7)

$$\begin{array}{rcll} 8A + 7B + C & = & 7 \\ 4A + 5B + C & = & 1 \\ \hline 4A + 2B & = & 6 \\ 2A + B & = & 3 \dots\dots (8) \end{array}$$

Dari (5) dan (6)

$$\begin{array}{rcll} 4A + 5B + C & = & 1 \\ 2A + 3B + C & = & -1 \\ \hline 2A + 2B & = & 2 \\ A + B & = & 1 \dots\dots (9) \end{array}$$

Dari (8) dan (9)

$$\begin{array}{rcll} 2A + B & = & 3 \\ A + B & = & 1 \\ \hline A & = & 2 \end{array}$$

Diperoleh

$$\begin{aligned} \Rightarrow A + B &= 1 \\ (2) + B &= 1 \\ B &= -1 \end{aligned}$$

$$\begin{aligned} \Rightarrow 4A + 5B + C &= 1 \\ 4(2) + 5(-1) + C &= 1 \\ 8 - 5 + C &= 1 \\ C &= -2 \end{aligned}$$

$$\begin{aligned} \Rightarrow 2A + B + C + D &= 2(2) + (-1) + (-2) + D = 1 \\ 4 - 1 - 2 + D &= 1 \\ D &= 0 \end{aligned}$$

$$\begin{aligned} \blacktriangleright \text{Suku Umum : } U_x &= f(x) + a \cdot 2^x \\ &= -x^2 - 2x + 2 \cdot 2^x \\ &= 2^{x+1} - x^2 - 2x \end{aligned}$$

$$\begin{aligned} \sum_{x=1}^n U_x &= \Delta^{-1} U_x \Big|_1^{n+1} \\ &= \Delta^{-1} 2^{x+1} - x^2 - 2x \Big|_1^{n+1} \\ &= 2^{x+1} - \frac{1}{3} x^{(3)} - (x)^{(2)} \Big|_1^{n+1} \end{aligned}$$

$$\begin{aligned}
&= \left[2^{n+2} - \frac{1}{3} (n+1)^{(3)} - (n+1)^{(3)} - \left(2^2 - \frac{1}{3} (1)^{(3)} - (1)^{(3)} \right) \right] \\
&= 2^{n+2} - \frac{1}{3} (n+1)n(n-1) - (n+1)n - 4 + 0 + 1 \\
&= 2^{n+2} - \frac{1}{3} n(n+1)(n-1) - n(n+1) - 3 \\
&= 2^{n+2} - \frac{1}{6} [2n(n^2-1) - 6n^2 - 6n - 3] \\
&= 2^{n+2} - \frac{1}{6} [2n^3 - 2n - 6n^2 - 6n - 3] \\
&= 2^{n+2} - \frac{1}{6} [2n^3 - 6n^2 - 8n - 3]
\end{aligned}$$

$$\begin{aligned}
17). \sum_{x=1}^n \frac{1}{4x^2-1} &= \sum_{x=1}^n \frac{1}{(2x-1)(2x+1)} \\
&= \frac{1}{2} \left(\frac{1}{2x-1} - \frac{1}{2x+1} \right) \\
&= \frac{1}{4} \left(\frac{1}{x-\frac{1}{2}} - \frac{1}{x+\frac{1}{2}} \right) \\
&= -\frac{1}{4} \left(\frac{1}{x+\frac{1}{2}} - \frac{1}{x-\frac{1}{2}} \right)
\end{aligned}$$

misal : $f(x) = \frac{1}{x^{(n)} - \frac{1}{2}} \Rightarrow f(x+1) = \frac{1}{x^{(n)} + \frac{1}{2}}$

$$\Rightarrow \Delta f(x) = \frac{1}{x^{(n)} + \frac{1}{2}} - \frac{1}{x^{(n)} - \frac{1}{2}}$$

Sehingga, didapatkan suku umumnya : $U_x = -\frac{1}{4} \left(\frac{1}{x^{(n)} + \frac{1}{2}} - \frac{1}{x^{(n)} - \frac{1}{2}} \right)$

$$\begin{aligned}
\text{Jumlah deret : } \sum_{x=1}^n \frac{1}{4x^2-1} &= \Delta^{-1} \frac{1}{4} \left(\frac{1}{x^{(n)} + \frac{1}{2}} - \frac{1}{x^{(n)} - \frac{1}{2}} \right) \Big|_1^{n+1} \\
&= -\frac{1}{4} \cdot \Delta^{-1} \cdot \Delta f(x) \Big|_1^{n+1} \\
&= -\frac{1}{4} f(x) \Big|_1^{n+1} \\
&= -\frac{1}{4} \left[\frac{1}{x - \frac{1}{2}} \right]_1^{n+1}
\end{aligned}$$

$$= -\frac{1}{4} \left[\frac{1}{(n+1) - \frac{1}{2}} - \frac{1}{1 - \frac{1}{2}} \right]$$

$$= -\frac{1}{4} \left[\frac{1}{n + \frac{1}{2}} - \frac{1}{\frac{1}{2}} \right]$$

$$= -\frac{1}{4} \left[\frac{2}{2n+1} - \frac{2}{1} \right]$$

$$= -\frac{1}{4} \left[\frac{2 - 4n - 2}{2n+1} \right]$$

$$= -\frac{1}{4} \left[\frac{-4n}{2n+1} \right]$$

15). $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots =$

Suku Umum : $U_x = \frac{1}{(3x+1)(3(x+1)+1)} = \frac{1}{(3x+1)(3x+4)}$

$$\begin{aligned} \ast \frac{1}{(3x+1)(3x+4)} &= \frac{A}{3x+1} + \frac{B}{3x+4} = \frac{1/3}{(3x+1)} - \frac{1/3}{(3x+4)} \\ &= \frac{1}{3} \left[\frac{1}{3x+1} - \frac{1}{3x+4} \right] \\ &= \frac{1}{9} \left[\frac{1}{x + \frac{1}{3}} - \frac{1}{x + \frac{4}{3}} \right] \end{aligned}$$

$$3Ax + 4A + 3Bx + B = 1$$

$$3A + 3B = 0$$

$$A = -B$$

$$4A + B = 1$$

$$4A - A = 1$$

$$A = \frac{1}{3}$$

Anggap $f(x) = \frac{1}{x + \frac{1}{3}} \rightarrow f(x+1) = \frac{1}{x + \frac{4}{3}}$

$$\begin{aligned} \Delta f(x) &= \frac{1}{x + \frac{4}{3}} - \frac{1}{x + \frac{1}{3}} \Rightarrow U_x = -\frac{1}{9} \left[\frac{1}{x + \frac{4}{3}} - \frac{1}{x + \frac{1}{3}} \right] \\ &= -\frac{1}{9} \Delta f(x) \end{aligned}$$

Deret

$$\begin{aligned} \sum_{x=0}^n U_x &= \sum_{x=0}^n -\frac{1}{9} \Delta f(x) = -\frac{1}{9} \Delta^{-1} \cdot \Delta f(x) \Big|_0^{n+1} \\ &= -\frac{1}{9} [f(x)]_0^{n+1} \\ &= -\frac{1}{9} \left[\frac{1}{x + \frac{1}{3}} \right]_0^{n+1} \end{aligned}$$

$$\Rightarrow -\frac{1}{9} \left[\frac{1}{n + \frac{4}{3}} - \frac{1}{\frac{1}{3}} \right] = \frac{1}{3} - \frac{1}{9n+12} = \frac{n+1}{3n+4}$$